Student's name ________________ Student's number ________________ Teacher's name ________________

PLC PRESBYTERIAN LADIES' COLLEGE SYDNEY 1888

2010
TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120

- Attempt questions 1-10
- All questions are of equal value

1	2	3	4	5	6	7	8	9	10	Total	Total
										/120	%

Question 1 **(12 marks)** **Start a new sheet of writing paper.**

Marks

a) Evaluate $\dfrac{e^5}{(-2)^2}$ correct to 3 significant figures. **2**

b) Simplify $\dfrac{x-1}{1-x}$ **1**

c) Solve $|2x-1|\geq 6$ and graph your solution on a number line. **3**

d) Given $\dfrac{2}{\sqrt{3}-2} = a\sqrt{3}+b$, find the values of a and b. **2**

e) The velocity of a particle is given by the equation $v=\dfrac{\log_e(t-1)}{2}$, metres per second. **2**

Find the acceleration at $t = 2$ seconds.

f) Factorise fully $x^6 - 64$. **2**

End of Question 1

Question 2 **(12 marks)** **Start a new sheet of writing paper.** **Marks**

a) Differentiate, with respect to x :

i) $\dfrac{\sin x}{x}$ **2**

ii) $\log_e (5x-4)^3$ **2**

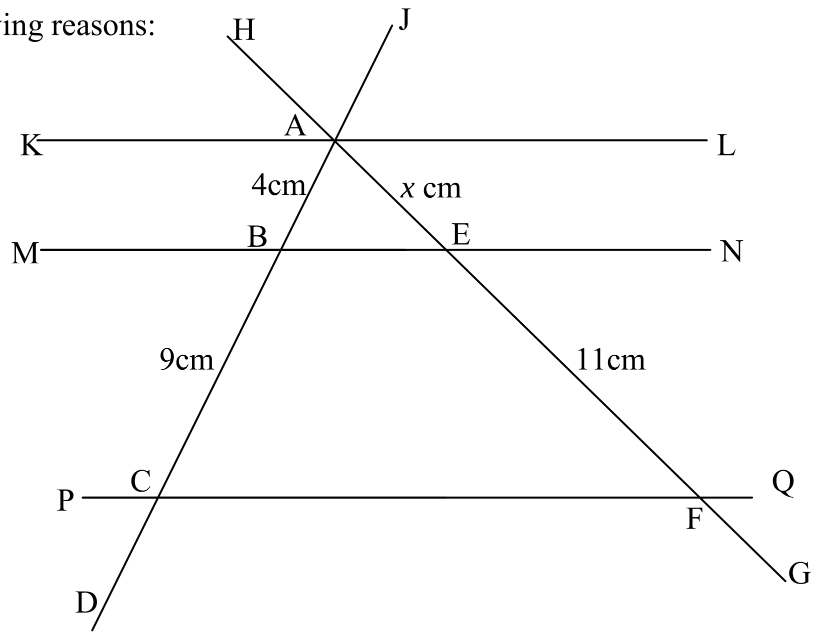
b) Find $\int 2e^{3x-5}\, dx$. **1**

c) Evaluate $\int_1^2 \dfrac{3x}{4x^2-2}\, dx$ leaving your answer in exact form. **3**

d) Find the exact value of $\sin 495° \tan 240°$ **2**

e) If KL, MN and PQ are all parallel lines, **2**

find x giving reasons:



End of Question 2

Question 3 **(12 marks)** **Start a new sheet of writing paper.**

Marks

$A(2,6)$ and $B(-3,1)$ are points on the number plane.

a) On your answer sheet, plot the points on a number plane (at least one-third of a page). **1**

b) Find the coordinates of C, the midpoint of AB. **1**

c) Find the gradient of the line AB. **1**

d) Show that the equation of AB is $y = x + 4$. **1**

e) Show that the equation of the perpendicular bisector to AB is $x + y - 3 = 0$. **2**

f) Show that $D(6,-3)$ lies on the perpendicular bisector. **1**

g) Find the coordinates of E such that $ADBE$ is a rhombus. **1**

h) Show that the perpendicular distance from D to AB is $\frac{13\sqrt{2}}{2}$ units. **2**

i) Find the distance AB in surd form. **1**

j) Hence, or otherwise, find the area of $ADBE$. **1**

End of Question 3

Question 4 **(12 marks)** **Start a new sheet of writing paper.**

Marks

a) Solve $2\sin\theta + \sqrt{3} = 0$ for $0 \le \theta \le 2\pi$ **2**

b) Find the value(s) of k in $x^2 - kx + 3k - 8 = 0$ if:

i) 2 is a root of the quadratic. **1**

ii) The roots are equal in magnitude but opposite in sign. **1**

iii) The roots are real. **3**

iv) The roots are reciprocals of one another. **1**

c)

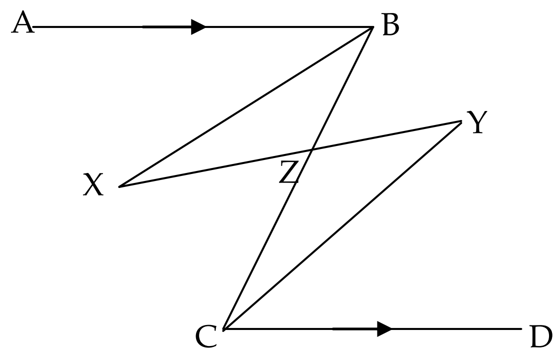


DIAGRAM NOT DRAWN TO SCALE

In the diagram above, AB is parallel to CD. XB bisects $\angle ABC$ and YC bisects $\angle BCD$. $BX = CY$,

i) Copy the diagram into your answer booklet, showing all information. **1**

ii) Prove that Z is the midpoint of BC. **3**

End of Question 4

Question 5 (12 marks) **Start a new sheet of writing paper.** **Marks**

a) If the sum of n terms of the series 15+13+11+… is 55, find the number of terms possible in the series. **2**

b) i) Find the sum of the series: $4+8+16+...+1024$ **2**

ii) Hence, or otherwise, simplify $2^4 \times 2^8 \times 2^{16} \times .... \times 2^{1024}$, leaving your answer in index form. **1**

c) i) Find the vertex of the parabola with focus at $(3,2)$ and directrix at $x=5$ **1**

ii) Hence, or otherwise, state the equation of the parabola. **1**

iii) Show that the points of intersection of the parabola and the line $2x+y-6=0$ are $(3,0)$ and $(0,6)$. **2**

iv) Find the area between the parabola and the line in the first quadrant. **3**

End of Question 5

Question 6 **(12 marks)** **Start a new sheet of writing paper.** **Marks**

If $f(x) = 6x^3 + 9x^2 - 3$,

a) i) Show that $6x^3 + 9x^2 - 3 = 3(x+1)^2(2x-1)$ **1**

ii) Hence find the x intercepts. **2**

b) Determine the y intercept. **1**

c) Find the stationary point(s) and determine their nature. **4**

d) Find the point(s) of inflexion. **2**

e) On a number plane (at least one-third of a page) sketch the curve **2**

$f(x) = 6x^3 + 9x^2 - 3$

showing all of the above features.

End of Question 6

Question 7 (12 marks) **Start a new sheet of writing paper.** **Marks**

a) Find the equation of the tangent to the curve $y = x^2 + \frac{1}{x}$ at the point where $x = 1$ (answer in gradient-intercept form) **3**

b) The velocity, v *m/s*, of a particle moving along a straight line is given by $v = 2 + t - 3t^2$ where t is in seconds. If the particle is initially at the origin,

i) Find when the particle is at rest **2**

ii) Sketch the velocity function from $t = 0$ to $t = 4$ seconds. **2**

iii) Find the total distance travelled by the particle in the first 4 seconds. **3**

c) The graph of $y = f'(x)$ is given below:

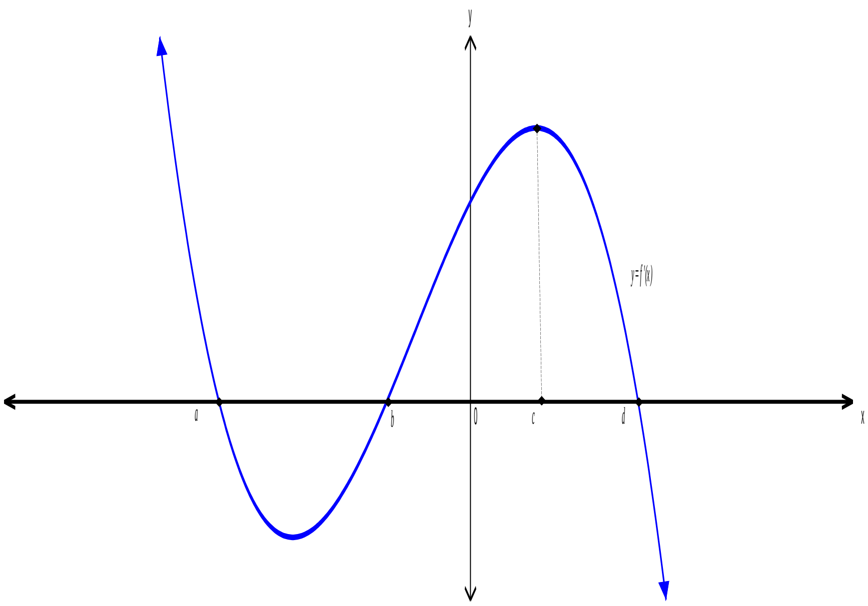


DIAGRAM NOT DRAWN TO SCALE

Copy or trace the curve into your answer booklet and carefully sketch and label the graph of $y = f(x)$ on the same set of axes. You may need to highlight your answer. **2**

End of Question 7

Question 8 **(12 marks)** **Start a new sheet of writing paper.**

Marks

a) i) Sketch $y = 1 - \sin 2x$ for $-\pi \leq x \leq \pi$ **2**

ii) Show that the equation of the tangent to the curve $y = 1 - \sin 2x$ at $x = 0$ is $2x + y = 1$. **2**

iii) For what range of values of m does the equation $1 - \sin 2x = 1 - mx$ have exactly 3 solutions in the domain $-\pi \leq x \leq \pi$. **2**

b) Find the volume generated when the area between the curve $y = \tan x$ and $x = 0$ and $x = \frac{\pi}{4}$ is rotated about the x-axis. **3**

c) Constance is doing yo-yo tricks. She is doing “around the world”. That is, she is keeping the yo-yo at its maximum length for a complete revolution. The length of the string on the yo-yo is 0.4 metres.

i) Find the distance the yo-yo travels from a point A to another point B if the angle subtended at the centre is $\frac{3\pi}{5}$ radians. **1**

ii) Find the area of the sector that the yo-yo sweeps around from A to B. **2**

End of Question 8

Question 9 **(12 marks)** **Start a new sheet of writing paper.** **Marks**

a) Simplify: **1**

$\tan\theta\sqrt{1-\sin^2\theta}$

b) Evaluate $\sum_{n=0}^{4}\sin^2\left(\frac{n\pi}{3}\right)$. **3**

c) Using Simpson's Rule with 5 function values, find the area bounded by the curve $y = 2^x$, the x-axis, x = -1 and x = 1. Leave your answer in surd form. **3**

d) A radioactive element used in a hospital decays according to $y = Ae^{-kt}$, where k is a constant and t is time in years. If the element has a mass, y, of 200g in 2009, and 182g in 2010:

i) Show that $k = -\log_e\left(\frac{91}{100}\right)$ **2**

ii) Find its mass in 2039. Answer to 1 decimal place. **1**

iii) Find its half-life (that is, the length of time elapsed when it has lost half of its original mass). **2**

End of Question 9

Question 10 **(12 marks)** **Start a new sheet of writing paper.** **Marks**

a) Solve: **3**

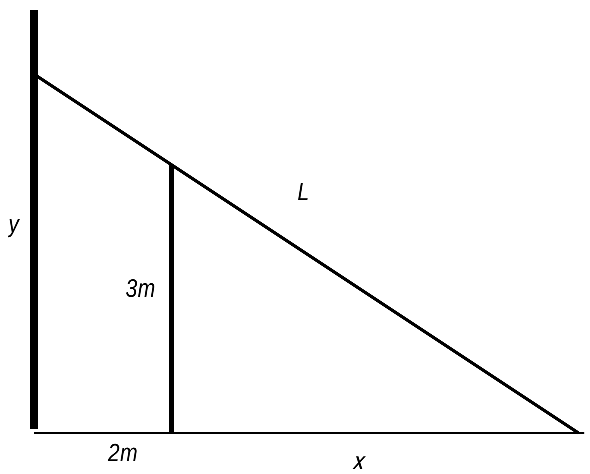
$\log_e(6x+9) - \log_e(x-1) = \log_e(3x-1)$

b) The general term of a geometric series is defined by $T_n = (x-2)^n$.
find:

i) The value(s) of x for which the series has a limiting sum **1**

ii) This limit in terms of x **2**

c) A 3m vertical fence stands 2 metres from a high vertical wall. A ladder is placed from the horizontal ground to the wall, resting on the fence. The base of the ladder is x metres from the fence.



i) Show that the square of the length of the ladder is given by **2**

$L^2 = (x+2)^2\left(1+\dfrac{9}{x^2}\right)$

ii) How long is the shortest ladder that can reach from the ground outside the fence to the wall, correct to 2 decimal places? Show all working. **4**

End of Examination

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \qquad x > 0$

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Question 1:

a) $\frac{e^5}{(-2)^2} = 37.103\ldots$

$= 37.1$ (3 sig figs)

b) $\frac{x-1}{1-x} = \frac{x-1}{-(-1+x)}$

$= \frac{(x-1)}{-(x-1)}$

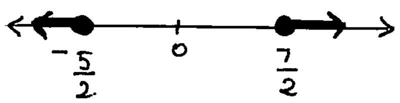
$= -1$

c) $|2x-1| \geq 6$

$2x-1 \leq -6, \quad 2x-1 \geq 6$

$2x \leq -5, \quad 2x \geq 7$

$x \leq -\frac{5}{2}, \quad x \geq \frac{7}{2}$



d) $\frac{2}{\sqrt{3}-2} = a\sqrt{3}+b$

LHS $= \frac{2}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2}$

$= \frac{2\sqrt{3}+4}{3-4}$

$= -2\sqrt{3}-4$

$\therefore -2\sqrt{3}-4 = a\sqrt{3}+b$

$\therefore a = -2, \quad b = -4$

e) $v = \frac{\log_e(t-1)}{2}$

$a = \frac{1}{2}\left(\frac{1}{t-1}\right)$

$a = \frac{1}{2(t-1)}$

when $t = 2$

$a = \frac{1}{2(1)}$

$a = \frac{1}{2}$ m/s^2

f) $x^6 - 64 = (x^3)^2 - 8^2$

$= (x^3-8)(x^3+8)$

$= (x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$

OR

$x^6 - 64 = (x^2)^3 - 4^3$

$= (x^2-4)(x^4+4x^2+16)$

$= (x-2)(x+2)(x^4+4x^2+16)$

Question 2:

a) i) $\frac{d}{dx}\left(\frac{\sin x}{x}\right)$

$u = \sin x$
$u' = \cos x$
$v = x$
$v' = 1$

$= \frac{vu' - uv'}{v^2}$

$= \frac{x\cos x - \sin x(1)}{x^2}$

$= \frac{x\cos x - \sin x}{x^2}$

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a) ii) $\frac{d}{dx} \log_e (5x-4)^3$

$= \frac{3(5x-4)^2(5)}{(5x-4)^3}$

$= \frac{15}{(5x-4)}$

OR $\frac{d}{dx} \log_e (5x-4)^3$

$= \frac{d}{dx} 3\log_e(5x-4)$

$= 3 \times \frac{5}{(5x-4)}$

$= \frac{15}{(5x-4)}$

b) $\int 2e^{3x-5}\,dx = \frac{2e^{3x-5}}{3} + c$

c) $\int_1^2 \frac{3x}{4x^2-2}\,dx = \frac{3}{8}\int_1^2 \frac{8x}{4x^2-2}\,dx$

$= \frac{3}{8}\left[\log_e(4x^2-2)\right]_1^2$

$= \frac{3}{8}\left[\log_e 14 - \log_e 2\right]$

$= \frac{3}{8}\log_e\left(\frac{14}{2}\right)$

$= \frac{3}{8}\log_e 7$

d) $\sin 495° \tan 240° = \frac{1}{\sqrt{2}} \times \sqrt{3}$

$= \frac{\sqrt{3}}{\sqrt{2}}$

or $\frac{\sqrt{6}}{2}$

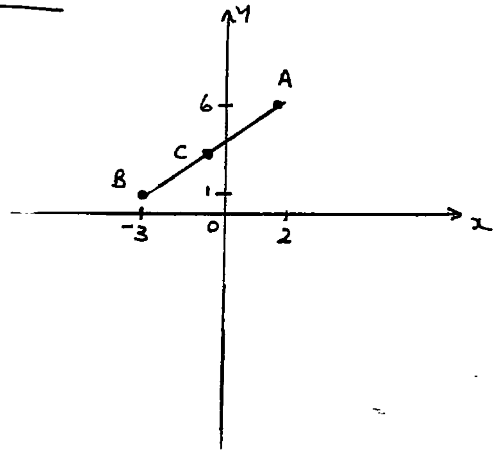
e) $\frac{AB}{BC} = \frac{AE}{EF}$ (when 3 or more lines are cut by two transversals, the ratio of intercepts are equal.)

$\frac{4}{9} = \frac{x}{11}$

$x = \frac{44}{9}$

Question 3:

a)



b) $C\left(-\frac{1}{2}, \frac{7}{2}\right)$

c) $m_{AB} = \frac{rise}{run}$

$m_{AB} = \frac{5}{5}$

$\therefore m_{AB} = 1$

d) $y - 6 = 1(x-2)$

$y - 6 = x - 2$

$y = x + 4$

e) $m_\perp = -1$ (since $m_1 m_2 = -1$)

& midpt $\left(-\frac{1}{2}, \frac{7}{2}\right)$

$\therefore$ equation: $y - \frac{7}{2} = -1\left(x + \frac{1}{2}\right)$

$2y - 7 = -2x - 1$

$2x + 2y - 6 = 0$

$x + y - 3 = 0$

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f) $D(6,-3)$ $\quad x+y-3=0$

$LHS = 6-3-3$
$= 0$
$= RHS$

$\therefore (6,-3)$ lies on perpendicular bisector.

g) $(-7, 10)$

h) $d_\perp = \dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

AB : $x-y+4$ $\quad D(6,-3)$

$\therefore d_\perp = \dfrac{|6-(-3)+4|}{\sqrt{1^2+(-1)^2}}$

$= \dfrac{13}{\sqrt{2}}$

$= \dfrac{13}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{13\sqrt{2}}{2}$ units

i) $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-3-2)^2+(1-6)^2}$

$= \sqrt{25+25}$

$= 5\sqrt{2}$ units

j) $A = \left(\frac{1}{2} \times \frac{13\sqrt{2}}{2} \times 5\sqrt{2}\right) \times 2$

$A = 65$ units^2

Question 4:

a) $2\sin\theta + \sqrt{3} = 0$

$\sin\theta = -\dfrac{\sqrt{3}}{2}$

$\theta = \pi + \dfrac{\pi}{3}, \; 2\pi - \dfrac{\pi}{3}$

$\theta = \dfrac{4\pi}{3}, \; \dfrac{5\pi}{3}$

b) $x^2 - kx + 3k - 8 = 0$

i) 2 root $\Rightarrow x = 2$ satisfies

$2^2 - k(2) + 3k - 8 = 0$
$4 - 2k + 3k - 8 = 0$
$k - 4 = 0$
$k = 4$

ii) roots equal in magnitude, opposite in sign.

$\therefore$ Let roots be $\alpha, -\alpha$

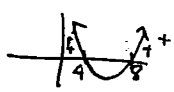
Sum of roots: $\alpha - \alpha = -\dfrac{b}{a}$

$0 = -\dfrac{(-k)}{1}$

$k = 0$

iii) real roots $\Rightarrow \Delta \geq 0$.

$b^2 - 4ac \geq 0$
$(-k)^2 - 4(1)(3k-8) \geq 0$
$k^2 - 12k + 32 \geq 0$
$(k-8)(k-4) \geq 0$



$k \leq 4, \; k \geq 8$

iv) reciprocal roots

Let roots be $\alpha, \dfrac{1}{\alpha}$

Product of roots $\alpha\left(\frac{1}{\alpha}\right) = \dfrac{c}{a}$

$1 = 3k - 8$

$k = 3$

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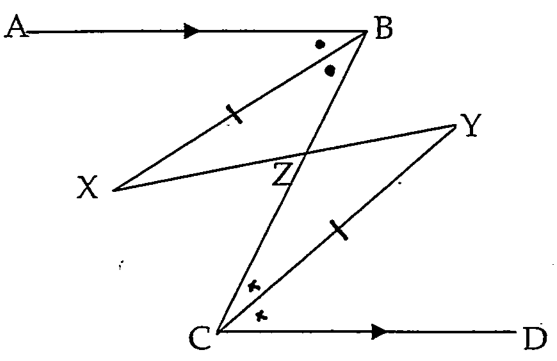
c) i)



ii) $\angle ABC = \angle BCD$ (alternate angles equal $AB \parallel CD$)

$\angle ABX = \angle XBZ$ (XB bisects $\angle ABC$)

$\angle ZCY = \angle YCD$ (CY bisects $\angle BCD$)

$\therefore \angle ABX = \angle XBZ = \angle ZCY = \angle YCD$

In $\triangle BXZ$ and $\triangle CYZ$,

$BX = CY$ (given)

$\angle XBZ = ZCY$ (proved above)

$\angle BZX = \angle CZY$ (vertically opposite)

$\therefore \triangle BXZ \equiv \triangle CYZ$ (AAS)

$\therefore BZ = CZ$ (corresponding sides in congruent triangles are equal).

$\therefore Z$ is midpoint of BC.

Question 5:

a) $15 + 13 + 11 + \dots + \dots$ $S_n = 55$, $a = 15$, $d = -2$.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$55 = \frac{n}{2}[30 + (n-1)(-2)]$

$110 = n[30 - 2n + 2]$

$110 = n[32 - 2n]$

$110 = 32n - 2n^2$

$2n^2 - 32n + 110 = 0$

$n^2 - 16n + 55 = 0$

$(n-11)(n-5) = 0$

$\therefore n = 5, 11$.

b) i) $4 + 8 + 16 + \dots + 1024$ $a = 4$, $r = 2$, $T_n = 1024$

$T_n = ar^{n-1}$

$1024 = 4(2)^{n-1}$

$256 = 2^{n-1}$

$2^8 = 2^{n-1}$

$\therefore n = 9$.

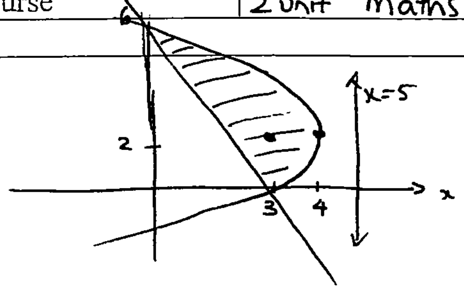
$S_n = \frac{a(r^n - 1)}{r-1}$

$= \frac{4(2^9 - 1)}{1}$

$= 2044$

ii) 2^{2044}

c) i)



$V(4,2)$

ii) eqn

$(y-2)^2 = -4(1)(x-4)$

$(y-2)^2 = -4(x-4)$

iii) $2x+y-6=0$

$y = -2x+6$.

subst. into parabola

$(y-2)^2 = -4(x-4)$

$\therefore (-2x+6-2)^2 = -4x+16$

$(-2x+4)^2 = -4x+16$

$4x^2 - 16x + 16 = -4x + 16$

$4x^2 - 12x = 0$

$4x(x-3) = 0$

$\therefore x=0, y=6$; $x=3, y=0$

iv) $A = \int_0^6 \text{parabola} - A_{\text{triangle}}$

$= \int_0^6 \text{parabola} - \frac{1}{2}\times 3\times 6$

$= \int_0^6 \text{parabola} - 9$

$(y-2)^2 = -4(x-4)$

$-\frac{1}{4}(y-2)^2 = x-4$

$x = 4 - \frac{1}{4}(y-2)^2$.

$\therefore \int_0^6 \left(4 - \frac{1}{4}(y-2)^2\right) dy - 9$

$= \left[4y - \frac{1}{4}\frac{(y-2)^3}{1\times 3}\right]_0^6 - 9$

$= \left(24 - \frac{1}{4}\left(\frac{64}{3}\right)\right) - \left(0 - \frac{1}{4}\left(\frac{-8}{3}\right)\right) - 9$

$= \left(24 - \frac{16}{3} - \frac{2}{3}\right) - 9$

$= 9$ units^2

Question 6:

a) i) show $6x^3+9x^2-3 = 3(x+1)^2(2x-1)$

RHS $= 3(x+1)^2(2x-1)$

$= 3(x^2+2x+1)(2x-1)$

$= (3x^2+6x+3)(2x-1)$

$= 6x^3+12x^2+6x-3x^2-6x-3$

$= 6x^3+9x^2-3$

$=$ LHS

$\therefore$ shown

ii) x-intercepts $\Rightarrow$ set $y=0$

$\therefore 6x^3+9x^2-3=0$

$3(x+1)^2(2x-1)=0$

$\therefore x = -1, \frac{1}{2}$

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b) y-intercept ⇒ set $x = 0$.

$\therefore y = 6(0)^3 + 9(0)^2 - 3$

$y = -3$

c) $f(x) = 6x^3 + 9x^2 - 3$

$f'(x) = 18x^2 + 18x$

for stat. pts $f'(x) = 0$

$\therefore 18x^2 + 18x = 0$

$18x(x+1) = 0$

$\therefore x = 0 \quad x = -1$.

$y = -3 \quad y = 0$

$\therefore (0, -3)$ & $(-1, 0)$ are stationary points

$f''(x) = 36x + 18$

when $x = 0$

$f''(0) = 18$

$\therefore$ concave up

$\therefore (0, -3)$ min stat. pt.

when $x = -1$

$f''(-1) = -18$

$\therefore$ concave down

$\therefore (-1, 0)$ max stat. pt.

d) for points of inflexion

$y'' = 0$

$\therefore 36x + 18 = 0$

$36x = -18$

$x = \frac{-18}{36}$

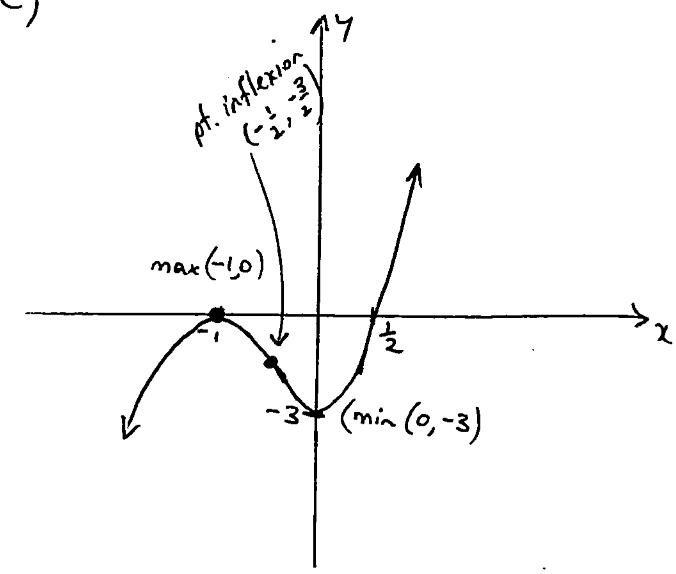
$x = -\frac{1}{2}$

x	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$
y''	-9	0	9

$\therefore$ concavity changes

$\therefore \left(-\frac{1}{2}, -\frac{3}{2}\right)$ is point of inflexion.

e)



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Question 7 :

a) $y = x^2 + \frac{1}{x}$

$y = x^2 + x^{-1}$

$\frac{dy}{dx} = 2x - x^{-2}$

at $x = 1$

$m_{tang} = 2 - 1^{-2}$

$= 2 - 1$

$\therefore m = 1$

at $x = 1$

$y = 1^2 + \frac{1}{1}$

$y = 2$

$(1, 2)$

$\therefore$ eqn of tangent :

$y - y_1 = m(x - x_1)$

$y - 2 = 1(x - 1)$

$y - 2 = x - 1$

$y = x + 1$

b) $v = 2 + t - 3t^2$

$t = 0 \quad x = 0$

i) at rest $v = 0$

$0 = 2 + t - 3t^2$

$3t^2 - t - 2 = 0$

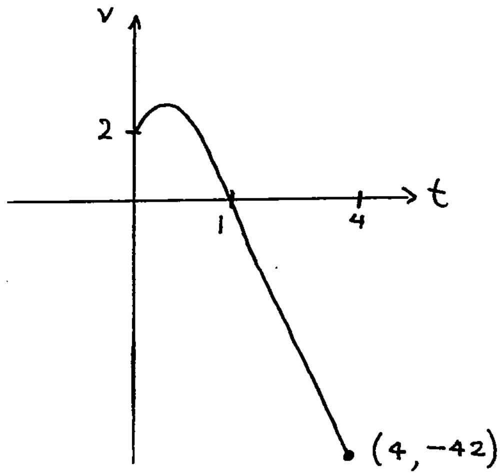
$(3t + 2)(t - 1) = 0$

$t = -\frac{2}{3}, \quad t = 1$

since $t \geq 0$

$\therefore t = 1$ sec.

ii)



iii) $v = 2 + t - 3t^2$

$x = \int (2 + t - 3t^2)\, dt$

$x = 2t + \frac{t^2}{2} - \frac{3t^3}{3} + c$

when $t = 0 \quad x = 0 \quad \therefore c = 0$

$\therefore x = 2t + \frac{1}{2}t^2 - t^3$

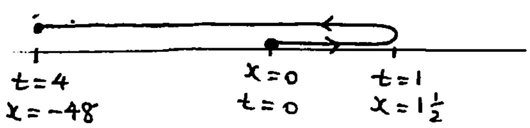
at $t = 0 \quad x = 0$

at $t = 1 \quad x = 2 + \frac{1}{2} - 1$

$= 1\frac{1}{2}$

at $t = 4 \quad x = 2(4) + \frac{1}{2}(4)^2 - (4)^3$

$= -48$

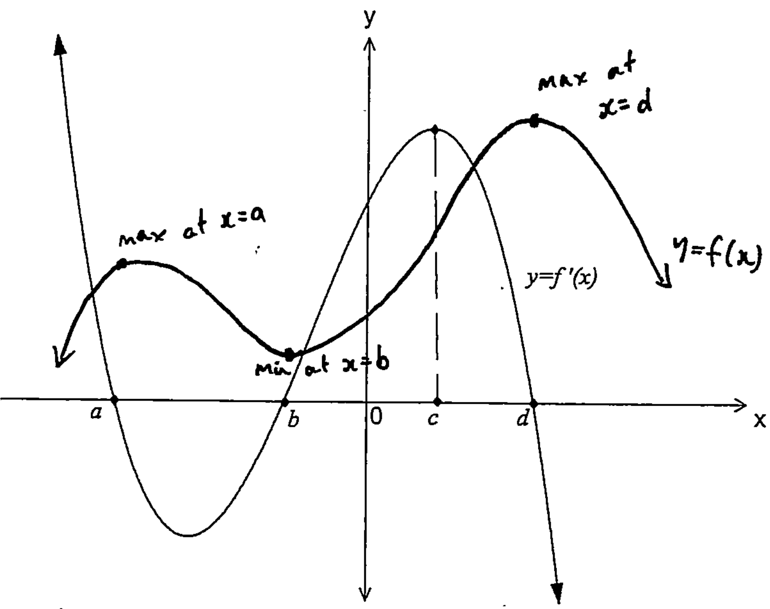


$\therefore$ total distance travelled

$= 1\frac{1}{2} + 1\frac{1}{2} + 48$

$= 51$ m

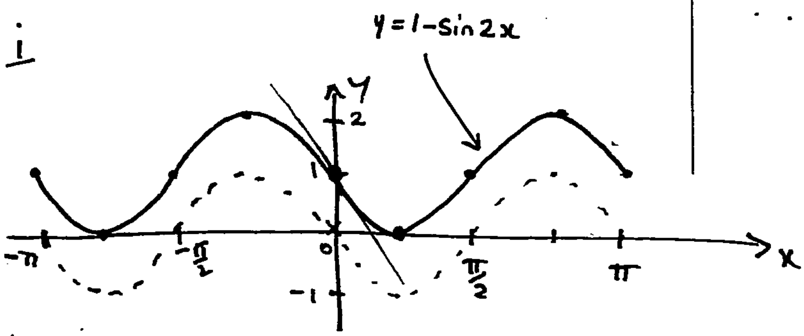
c)



Question 8 :

a) i) $y = 1 - \sin 2x \qquad -\pi \le x \le \pi$

a i



ii) $y' = -2\cos 2x$

at $x = 0$

$m_{tang} = -2\cos 0$

$= -2$

$\therefore y - 1 = -2(x - 0)$

$y - 1 = -2x$

$2x + y = 1$

iii) $0 < m < 2$

b) $V = \pi \int_a^b y^2 \, dx$

$= \pi \int_0^{\pi/4} \tan^2 x \, dx$

we know $\tan^2 x + 1 = \sec^2 x$

$\therefore \tan^2 x = \sec^2 x - 1$

$\therefore V = \pi \int_0^{\pi/4} (\sec^2 x - 1) \, dx$

$= \pi \left[\tan x - x \right]_0^{\pi/4}$

$= \pi \left[\left(\tan \frac{\pi}{4} - \frac{\pi}{4} \right) - (0 - 0) \right]$

$= \pi \left[1 - \frac{\pi}{4} \right]$

$= \frac{\pi}{4}(4 - \pi)$ units^3

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c) $r = 0.4$ m

i) $l = r\theta$

$= 0.4 \times \frac{3\pi}{5}$

$= 0.24\pi$ m or $\frac{6\pi}{25}$ m

ii) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}(0.4)^2\left(\frac{3\pi}{5}\right)$

$= 0.048\pi$ m^2

OR $\frac{6\pi}{125}$ m^2

Question 9:

a) $\tan\theta\sqrt{1-\sin^2\theta}$

$= \frac{\sin\theta}{\cos\theta}\sqrt{\cos^2\theta}$

$= \frac{\sin\theta}{\cos\theta} \times \cos\theta$

$= \sin\theta$

c) $A = \frac{h}{3}\left[y_0 + y_n + 4(y_1 + y_3) + 2(y_2)\right]$

$= \frac{\frac{1}{2}}{3}\left[2^{-1} + 2^1 + 4\left(2^{-\frac{1}{2}} + 2^{\frac{1}{2}}\right) + 2(2^0)\right]$

$= \frac{1}{6}\left[\frac{9}{2} + 6\sqrt{2}\right]$

$= \frac{3}{4} + \sqrt{2}$

b) $\sum_{n=0}^{4} \sin^2\left(\frac{n\pi}{3}\right)$

$= \sin^2(0) + \sin^2\left(\frac{\pi}{3}\right) + \sin^2\left(\frac{2\pi}{3}\right) + \sin^2\left(\frac{3\pi}{3}\right) + \sin^2\left(\frac{4\pi}{3}\right)$

$= 0 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 0 + \left(-\frac{\sqrt{3}}{2}\right)^2$

$= \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$

$= \frac{9}{4}$

d) i) $y = Ae^{-kt}$

2009 $t = 0$ $y = 200$

$200 = Ae^0$

$A = 200$

$y = 200e^{-kt}$

2010 $t = 1$ $y = 182$

$182 = 200e^{-k}$

$\frac{182}{200} = e^{-k}$

$\ln\left(\frac{182}{200}\right) = -k$

$k = -\ln\left(\frac{91}{100}\right)$

ii) when $t = 30$ $y = ?$

$y = 200e^{-(-\ln\frac{91}{100})30}$

$y = 11.8g$

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iii) $t = ?$ $y = 100$

$$100 = 200e^{-(-\ln\frac{91}{100})t}$$

$$\frac{1}{2} = e^{(\ln\frac{91}{100})t}$$

$$\ln\left(\frac{1}{2}\right) = \ln\left(\frac{91}{100}\right)t$$

$\therefore t = 7.349\ldots$ yrs

$t = 7.3$ years

Question 10:

a) $\ln(6x+9) - \ln(x-1) = \ln(3x-1)$

$$\therefore \ln\left(\frac{6x+9}{x-1}\right) = \ln(3x-1)$$

$$\therefore \frac{6x+9}{x-1} = 3x-1$$

$$\therefore 6x+9 = (3x-1)(x-1)$$

$$6x+9 = 3x^2 - 3x - x + 1$$

$$\therefore 3x^2 - 10x - 8 = 0$$

$$(3x+2)(x-4) = 0$$

$x = -\frac{2}{3}$, $x = 4$

but $x \neq -\frac{2}{3}$ as $(x-1) > 0$

$\therefore x = 4$ is only sol^n.

b) i) $T_n = (x-2)^n$

$r = (x-2)$

$\therefore -1 < x-2 < 1$

$1 < x < 3$

ii) $S_\infty = \frac{a}{1-r}$

$$= \frac{x-2}{1-(x-2)}$$

$$= \frac{x-2}{-x+3}$$

$$S_\infty = \frac{x-2}{3-x}$$

c) i) $L^2 = y^2 + (x+2)^2$ (by pyth. thm)

$$L^2 = (x+2)^2 + y^2$$

we know $\frac{3}{x} = \frac{y}{x+2}$ (similar Δs)

$$\therefore y = \frac{3(x+2)}{x}$$

$$\therefore L^2 = (x+2)^2 + \left[\frac{3(x+2)}{x}\right]^2$$

$$L^2 = (x+2)^2\left[1 + \frac{9}{x^2}\right]$$

ii) $L = (x+2)\sqrt{1+\frac{9}{x^2}}$

$$= (x+2)\left(1+\frac{9}{x^2}\right)^{\frac{1}{2}}$$

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$$\frac{dL}{dx} = (x+2)\,\tfrac{1}{2}\left(1+\frac{9}{x^2}\right)^{-\frac{1}{2}}\left(-18x^{-3}\right) + \left(1+\frac{9}{x^2}\right)^{\frac{1}{2}}(1)$$

$$\frac{dL}{dx} = \frac{(x+2)(-18)}{2\sqrt{1+\frac{9}{x^2}}\,(x^3)} + \frac{\sqrt{1+\frac{9}{x^2}}}{1}$$

for min $\frac{dL}{dx} = 0$

$$\therefore 0 = (x+2)(-18) + 2x^3\left(1+\frac{9}{x^2}\right)$$

$$18(x+2) = 2x^3\left(1+\frac{9}{x^2}\right)$$

$$\cancel{18x} + 36 = 2x^3 + \cancel{18x}$$

$$x^3 = 18$$

$$x = \sqrt[3]{18}$$

x	$\sqrt[3]{18}^-$	$\sqrt[3]{18}$	$\sqrt[3]{18}^+$
$\frac{dL}{dx}$	$-$	0	$+$

$\therefore$ min.

$$\therefore L = \left(\sqrt[3]{18}+2\right)\sqrt{1+\frac{9}{(\sqrt[3]{18})^2}}$$

$$L = 7.02 \text{ m } (2\text{ dp})$$